A Magnetic Vector Potential Volume Integral Formulation for Nonlinear Magnetostatic Problems

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This paper presents a novel volume integral formulation based on the interpolation of the magnetic vector potential on edge elements in order to deal with 3D nonlinear magnetostatic problems. The formulation ensures rigorously the solenoidality of magnetic induction. A strong point is that the convergence of the nonlinear resolution is easily reached after a few iterations without any relaxation. Computed results for the TEAM Workshop problem 13 and for an actuator demonstrate the efficiency and accuracy of this new formulation.

***Index Terms*— edge elements, integral formulation, nonlinear magnetostatic, volume integral method**

I. Introduction

VOLUME integral method (VIM) is known today as an interesting alternative to classical finite element method (FEM) for solving of nonlinear magnetostatic problems. The main advantage of VIM over FEM is that neither free space mesh nor boundary conditions are required, only active regions have to be meshed.

The integral formulations are the core of magnetostatic VIM. Different kinds of formulations have already been presented in the literature [1]–[6]. Although these formulations are able to yield accurate results in variety numerical examples, their applications to general magnetostatic problems are not without any difficulties. The magnetic moment method (MMM) [1] [2], which is the oldest and also the most popular formulation, cannot accurately deal with problems with high susceptibility material and can suffer from the well-known "looping pattern". More recently, **H**-edge and φ-nodal formulations were proposed in [3] [4] [5]. The **H**-edge formulation [3] is based on the interpolation of the magnetic field with edge elements. The φ-nodal formulation is on the other hand [4] [5] established by an interpolation of the scalar potential on nodal elements. They seem to be more flexible and robust than the MMM. However, these formulations present a quite poor convergence rate for nonlinear resolution due to the use of a **B**(**H**) curve. Thus, a relaxation process, which increases the computation time, is required in order to achieve the convergence. Most recently, the B-facet formulation based on interpolation of magnetic flux on facet elements has been proposed in [6]. This formulation is a very interesting approach and ensures a good convergence rate due to the use of a **H**(**B**) curve and results in an excellent accuracy even with very coarse meshes. Nevertheless, the B-facet formulation requires the use a face-tree in order to impose the solenoidality of **B**. This procedure can be time consuming and decreases the convergence rate of linear system resolutions.

We propose in this paper a useful formulation based on the interpolation of the magnetic vector potential on edge elements. We will demonstrate that this formulation can overcome the limits of the aforementioned integral formulations. The convergence of the nonlinear resolution is easily reached after a few iterations without any relaxation. Moreover, the fully dense integral matrix does not need to be recomputed during the nonlinear resolution and can be compressed thanks to efficient matrix compression algorithms such as the fast multipoles method or adaptive cross approximation.

II. Formulations

For a magnetostatic problem, the governing equations are $div\mathbf{B} = 0$ and $\mathbf{rotH} = \mathbf{J_0}$. By introducing the magnetization **M** so that $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, the constitutive equation is expressed either as $\mathbf{H} = \upsilon\mathbf{B}$ or $\mathbf{M} = (\upsilon_0 - \upsilon)\mathbf{B}$. The Maxwell-Ampère equation allows the writing of the magnetic field **H** as

$$\mathbf{H} = \mathbf{H}_0 - \mathbf{grad}\,\varphi_r\,, \tag{1}$$

where $\mathbf{H_0}$ is the magnetic field created by $\mathbf{J_0}$ in the vacuum, and φ_r is the reduced magnetic scalar potential created by magnetic material. The potential φ_r is determined by

$$\varphi_r = \frac{1}{4\pi}\int_\Omega \mathbf{M}\cdot\mathbf{grad}\left(\frac{1}{r}\right)\mathrm{d}\Omega\,, \tag{2}$$

where r is the distance between the observation and integration points, and Ω the magnetic domain.

The magnetic vector potential **A** defined by $\mathbf{B} = \mathbf{rotA}$ can be approximated by the interpolation of tangential components A_j on edge elements as $\mathbf{A} = \sum_{j=1}^{N}\mathbf{w}_j A_j$. in this expression; N is the number of edge elements and the $\mathbf{w}_j$ are first order shape functions of edge elements. Projecting (1) on $\mathbf{rotw}_i$ in domain Ω, we get

$$\int_\Omega \mathbf{rotw}_i\cdot\mathbf{H}\,\mathrm{d}\Omega + \int_\Omega \mathbf{rotw}_i\cdot\mathbf{grad}\,\varphi_r\,\mathrm{d}\Omega = \int_\Omega \mathbf{rotw}_i\cdot\mathbf{H}_0\,\mathrm{d}\Omega\,. \tag{3}$$

The two terms on the left hand can be developed as follows:

$$\mathbf{I}_1 = \int_\Omega \mathbf{rotw}_i\cdot\mathbf{H}\mathrm{d}\Omega = \int_\Omega \mathbf{rotw}_i\cdot\nu\,\mathbf{B}\mathrm{d}\Omega = \sum_{j=1}^{N}\left(\int_\Omega \mathbf{rotw}_i\cdot\nu\,\mathbf{rotw}_j\mathrm{d}\Omega\right)A_j; \tag{4}$$

$$\mathbf{I}_2 = \int_\Omega \mathbf{rotw}_i \cdot \mathbf{grad}\varphi_r \mathrm{d}\Omega = \int_\Omega div(\mathbf{rotw}_i \cdot \varphi_r)\mathrm{d}\Omega = \int_\Gamma \mathbf{rotw}_i \cdot \mathbf{n}\varphi_r \mathrm{d}\Gamma \,, \quad (5)$$

in which Γ is the boundary of domain Ω, **n** is normal of Γ. From (2) and the constitutive equation $\mathbf{M} = (\upsilon_0 - \upsilon)\mathbf{B}$, we express the potential φ_r as a function of the A_j. The term I_2 is then written as

$$\mathbf{I}_2 = \sum_{j=1}^{N} \left[\frac{1}{4\pi} \int_\Gamma (\mathbf{rotw}_i \cdot \mathbf{n}) \left(\int_\Omega (\nu - \nu_0)\, \mathbf{rotw}_j \cdot \mathbf{grad}\left(\frac{1}{r}\right) \mathrm{d}\Omega \right) \mathrm{d}\Gamma \right] A_j \,. \quad (6)$$

Hence, using (4) and (6), we write (3) in matrix form as

$$(\mathbf{R} + \mathbf{L})\, A = U_0 \,, \quad (7)$$

where matrices **R**, **L**, and vector U_0 are defined by

$$\mathbf{R}_{ij} = \int_\Omega \mathbf{rotw}_i \cdot \nu\, \mathbf{rotw}_j \mathrm{d}\Omega$$

$$\mathbf{L}_{ij} = \frac{1}{4\pi} \int_\Gamma (\mathbf{rotw}_i \cdot \mathbf{n}) \left(\int_\Omega (\nu - \nu_0)\, \mathbf{rotw}_j \cdot \mathbf{grad}\left(\frac{1}{r}\right) \mathrm{d}\Omega \right) \mathrm{d}\Gamma \quad (8)$$

$$U_{0i} = \int_\Omega \mathbf{rotw}_i \cdot \mathbf{H}_0 \mathrm{d}\Omega \,.$$

Since edge elements of first order are used, the magnetic field **B** is constant in each tetrahedral element. Thus, υ is constant piecewise and matrix **L** is determined by

$$\mathbf{L}_{ij} = \frac{1}{4\pi} \int_\Gamma (\mathbf{rotw}_i \cdot \mathbf{n}) \left(\sum_{k=1}^{N_f} \delta\nu_f \int_{\Gamma_f} \frac{\mathbf{rotw}_j \cdot \mathbf{n}}{r} \mathrm{d}\Gamma_f \right) \mathrm{d}\Gamma \,, \quad (9)$$

where N_f is number of facets and $\delta\upsilon_f$ the difference of reluctivity between two elements sharing the facet *f*.

Let us notice that $\mathbf{rotw}_i{\cdot}\mathbf{n}$ is a scalar quantity defined by $\pm 1/S_f$ where S_f is the surface of a facet element containing edge *i*. Thus, $\mathbf{rotw}_i{\cdot}\mathbf{n}$ is taken out of the integrand (9) and matrix **L** is decomposed into the product of two terms. The first is the vector δυ dealing with the material properties. The second is the fully populated matrix $\mathbf{L}_0$ which depends only on the double surface integration over faces i.e. properties of elements geometry. Thus, matrix $\mathbf{L}_0$ can be computed only once and during the nonlinear resolution matrix **L** is updated by the product of $\mathbf{L}_0$ and vector δυ, which is recomputed after each iteration. Matrix **R** is a FEM matrix structure.

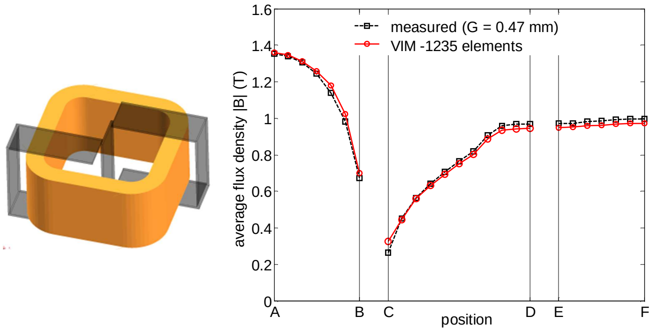
The singularity of Green's function in (9) is computed exactly by an analytical integration. Due to the use of the vector potential **A**, the solenoidality of magnetic induction is naturally verified.

System (7) is compatible according to [7]. Consequently, it can be straightforwardly solved by an iterative solver without any gauge condition and the obtained solution is unique.

III. Numerical Results

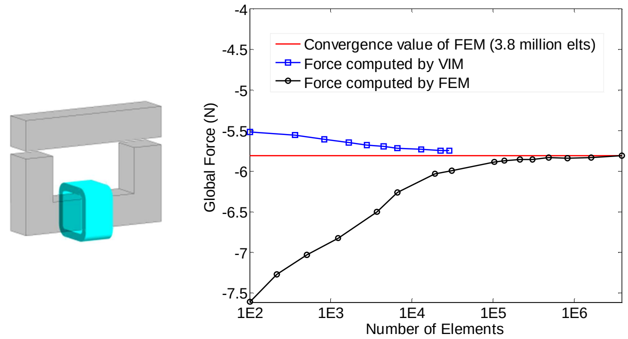
The TEAM Workshop problem 13 [8] (Fig.1.a) and an actuator (Fig.2.a) were studied in order to demonstrate the efficiency of the proposed formulation. The classic Newton Raphson method was used for solving of the nonlinear system.

For the problems13, the magnetomotive force is 1000 AT. The obtained average flux densities inside steel plate compared to measured values are shown on Fig.1.b. Computed



a) b)

Fig.1. TEAM 13. a) Geometry. b) Average flux density inside steel plate.



a) b)

Fig.2. Actuator. a) Geometry. b) Convergences of force computation

results are very close to the measurement. A very coarse mesh of 1,235 tetrahedral elements gives accurate results. With an absolute stop criterion of 1E-6, the Newton Raphson procedure has converged after five iterations in 6.7 seconds on a standard computer.

The goal of the second example was to compute the global force acting on the pallet of the actuator. Once (7) was solved, both the method of equivalent magnetic charges and virtual work can be applied. They provide the same results. Fig. 2.b shows the comparaison of the convergence with FEM. In order to obtain a result deviated by 5% from the reference value, VIM needs only a mesh of 100 tetrahedra whereas the FEM requires a mesh containing 12,000 elements on active regions.

IV. References

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